UMD-PP-99-109

Sterile Neutrinos in E_6 and a Natural Understanding of Vacuum Oscillation Solution to the Solar Neutrino Puzzle

Z. Chacko and R. N. Mohapatra
Department of Physics, University of Maryland, College Park, MD, 20742

(May, 1999)

Abstract

If Nature has chosen the vacuum oscillation solution to the Solar neutrino puzzle, a key theoretical challenge is to understand the extreme smallness of the $\Delta m^2_{\nu_e-\nu_X}$ ($\sim 10^{-10}$ eV^2) required for the purpose. We find that in a class of models such as $[SU(3)]^3$ or its parent group E_6, which contain one sterile neutrino, ν_{is} for each family, the $\Delta m^2_{\nu_i-\nu_{is}}$ is proportional to the cube of the lepton Yukawa coupling. Therefore fitting the atmospheric neutrino data then predicts the $\nu_e-\nu_{es}$ mass difference square to be $\sim \left(\frac{m_e}{m_\mu}\right)^3 \Delta m^2_{atmos}$, where the atmospheric neutrino data is assumed to be solved via the $\nu_\mu-\nu_{\mu s}$ oscillation. This provides a natural explanation of the vacuum oscillation solution to the solar neutrino problem.

I. INTRODUCTION

As is widely known by now, the Super-Kamiokande data has provided conclusive evidence for the existence of oscillations of the muon neutrinos from cosmic rays [1]. While it is yet to be determined what final state the cosmic ray ν_μs oscillate to (ν_τ or $\nu_{\mu s}$), it is known that the mixing angle is near maximal and the $\Delta m^2_{atmos} \sim 10^{-3}$ eV^2. Similarly the solar neutrino data from Super-Kamiokande and other experiments [2] are also making a very convincing case for oscillations of the electron neutrinos emitted by the Sun in order to understand the observed deficit of the solar neutrinos [3]. Again, in this case also, it is not clear what final state the ν_e oscillates to on its way from the Sun to Earth. It could either be ν_μ or ν_{es}. There are however several mixing angle and mass difference possibilities in the solar case [3]. One of the possibilities is the vacuum oscillation of $\nu_e - \nu_\mu$ or $\nu_e - \nu_{es}$ type. In order to explain the observations one needs in this case that the $\Delta m^2_{\nu_e-\nu_X} \sim 10^{-10}$ eV^2 and a maximal mixing like in the atmospheric neutrino case. The recent indications of a seasonal dependence of the solar neutrino events in the 708 day Super-Kamiokande data [4] would seem to support this explanation although it is by no means the only way to understand it [5]. If the vacuum oscillation explanation finally wins, then a serious theoretical challenge is to understand the unusually small mass difference squared between the neutrinos needed for the purpose. It is the goal of this letter to propose a way to answer this challenge within a gauge theory framework.

The first observation that motivates our final scenario is the symmetry between the solution to the atmospheric neutrino data and the vacuum oscillation solution to the solar neutrino data in that the mixing angles are maximal. This might suggest a generation independence of the neutrino mixings patterns. An implementation of such an idea would naturally require that in each case i.e. solar as well as atmospheric the active neutrinos (i.e. ν_e and ν_μ) oscillate into the sterile neutrinos [6] to be denoted by ν_{es} and $\nu_{\mu s}$ respectively. The complete three family picture would then require that there be one sterile neutrino per family. One class of models that lead to such a scenario [7] is the mirror universe [8] picture where the particles and the forces in the standard model are duplicated in a mirror symmetric manner. There is no simple way to understand the ultra small Δm^2 needed for the vacuum oscillation solution in this case. In this letter we focus on an alternative scheme based on the grand unification group $[SU(3)]^3$ or its parent group E_6.

We find it convenient to use the E_6 notation. As is well-known [9], under the SO(10) group, the **27**-dimensional representation of E_6 decomposes to $\mathbf{16}_{+1} \oplus \mathbf{10}_{-2} \oplus \mathbf{1}_{+4}$ where the subscripts represent the U(1) charges. The **16** is well known to contain the left and the right handed neutrinos (to be denoted by us as ν_i and ν_i^c, i being the family index). The **10** contains two neutral colorless fermions which behave like neutrinos but are $SU(2)_L$ doublets and the last neutral colorless fermion in the **27**, which we identify as the sterile neutrino is the one contained in **1** (denoted by ν_{is}). In general in this model, we will have for each generation a 5×5 "neutrino" mass matrix and we will show how the small masses for the sterile neutrino and the known neutrino come out as a consequence of a generalized seesaw mechanism. Furthermore, we will see how as a consequence of the smallness of the Yukawa couplings of the standard model, we will not only get maximal mixing between the active and the sterile neutrinos of each generation but also the necessary ultra-small Δm^2 needed in the vacuum oscillation solution without fine tuning of parameters[1]. The way this comes about in our model is that to the lowest order in the Yukawa couplings, the ν_i and ν_{is} form a Dirac neutrino with a mass proportional to the generational Yukawa coupling λ_i of the corresponding generation. However they become pseudo-Dirac to order λ_i^3 leading to nearly degenerate neutrinos with a mass splitting $\Delta m_i^2 \approx \lambda_i^3$. Therefore fixing the Δm^2_{atmos} gives the right value for the Δm^2_e needed for the vacuum oscillation solution.

Let us now present the basic idea of the model for one generation of neutrinos consisting of ν_i, ν_{is}. Suppose that their mass matrix is given by the following 2×2 matrix:

$$M_i = m_{0i} \begin{pmatrix} \lambda_i^2 & \lambda_i \bar{f}_i \\ \lambda_i \bar{f}_i & \lambda_i^2 \bar{\epsilon}_i \end{pmatrix} \tag{1}$$

Since $\lambda_i \ll 1$, it is clear that the two neutrinos are maximally mixed with a mass $m_i \simeq \lambda_i \bar{f}_i m_{0i}$ amd $\Delta m_i^2 \simeq \lambda_i^3 \bar{f}_i m_{0i}^2$ provided $\bar{\epsilon} \leq 1$ and $\bar{f} \geq 1$. These relations are true generation by generation. We shall show in the next section that a mass matrix of this form emerges naturally from E_6 and its subgroup $[SU(3)]^3$ with $\bar{f}_i \approx 1$. The main difference between

[1] An E_6 model for the neutrino puzzles was first discussed by Ma [10], where his goal was to understand the smallness of the sterile neutrino masses. Our model is different in many respects and addresses the question of maximal mixing, small Δm^2's as well as the small neutrino masses. Our picture also differs from other recently proposed models [11]

these two groups arises from the fact that for the simplest models based on E_6 the Yukawa coupling λ_i is necessarily related to the Yukawa coupling of the corresponding up type quark. This is not true for $[SU(3)]^3$, for which λ_i is expected to be related to the Yukawa coupling of the corresponding lepton.

Now let us look at the atmospheric neutrino data. Since $\Delta m_2^2 \simeq 2-5\times 10^{-3}$ eV^2, if we choose the ν_μ mass to be of order $0.2-0.5$ eV (anticipating that we want to accomodate the LSND data [12]), then we get $\lambda_2 \simeq 10^{-2}$, which is a typical second generation Yukawa coupling. Note that this is a plausible value even for $[SU(3)]^3$ since in supersymmetric models $m_\mu \simeq \lambda_2 v_d$ and v_d can be considerably less (e.g 10 GeV for a $tan\beta \simeq 24$) than the standard model value of 246 GeV for the symmetry breaking parameter. Since the same formula applies to the ν_e and ν_{es} sector, assuming no large flavour dependence in the coefficients f_i and ϵ_i, all we need in order to predict their masses and mass differences is the value for λ_1/λ_2. Irrespective of whether we consider E_6 or $[SU(3)]^3$ we find $\lambda_1/\lambda_2 \simeq 5\times 10^{-3}$. This leads to a value for the $\Delta m_1^2 \simeq 2\times 10^{-10}$ eV^2, which is clearly of the right order of magnitude. Our main point is not to insist on precise numbers but rather to illustrate the idea that a cubic dependence of the neutrino mass difference squares on the generational Yukawa couplings to leptons of the standard model can lead to an understanding of the extreme smallness of Δm^2 value needed in the vacuum oscillation solution to the solar neutrino puzzle [13].

Extending our idea to the third generation, we find that value of the ν_τ mass is $(\lambda_3/\lambda_2)0.2$ eV. This imples $m_{\nu_\tau} \approx 2-3$ eV for $[SU(3)]^3$ which is interesting for cosmology since this would mean that about 10-15% of the mass of universe could come from neutrinos. This expectation can eventually be tested when the finer measurements of the angular power spectrum is carried out in the MAP and PLANCK experiments in the next few years. However for minimal versions of E_6, $\lambda_3/\lambda_2 \sim m_t/m_c$ yielding a value $m_{\nu_\tau} \approx 20-30$ eV which is unacceptable for a realistic cosmology. This means that the simplest E_6 model that can accomodate our scenario is one where the quark lepton symmetry is broken. The other possibility is to have some flavour dependence on the coefficients $\bar{f}_i$ and m_{0i}. The extent of required flavour dependence is certainly not extreme and we consider models based on both groups as realistic candidates for a complete theory of neutrino masses.

II. THE MODEL

Let us now proceed to construct the mass matrix in Eq. (1) in the context of an E_6 model. As usual, we will assign matter to the **27** dimensional representation of the group and we have already noted that there are five neutrino-like fields in the model which will mix among each other subsequent to symmetry breaking. It is therefore necessary to describe the symmetry breaking of E_6. To implement the symmetry breaking we use three pairs of $\mathbf{27}+\bar{\mathbf{27}}$ representations and one **78**-dim. field. The pattern of symmetry breaking is as follows:

1) $< 27_1 >$ and $< \overline{27}_1 >$ have GUT scale vevs in the SO(10) singlet direction.

2) $< 27_{16} >$ and $< \overline{27}_{16} >$ have GUT scale vevs in the ν and ν^c directions respectively. They break SO(10) down to SU(5).

3) The $< 78_{[1,45]} >$ completes the breaking of SU(5) to the standard model gauge group at the GUT scale. We assume the VEVs reside both in the adjoint and in the singlet of SO(10).

4) $< 27_{10} >$ and $< \overline{27}_{10} >$ contain the Higgs doublets of the MSSM. It is assumed that H_u and H_d are both linear combinations arising partially from the $< 27_{10} >$ and partially from the $< \overline{27}_{10} >$

In addition to the above there is another field labelled by $27'$ whose ν^c component mixes with a singlet S and one linear combination of this pair (denoted by S') remains light below the GUT scale. As a consequence of radiative symmetry breaking this picks up a VEV at the electroweak scale. We will show later how this can occur. The remaining components of $27'$ have GUT scale mass.

Let us now write down the relevant terms in the superpotential that lead to a 5×5 "neutrino" mass matrix of the form we desire. To keep matters simple let us ignore generation mixings, which can be incorporated very trivially.

$$W \;=\; \lambda_i\psi_i\psi_i 27_{10} + f_i\psi_i\psi_i 27' + \frac{\alpha_i}{M_{P\ell}}\psi_i\psi_i 27_1 78_{[1,45]} + \frac{\gamma_i}{M_{P\ell}}\psi_i\psi_i\overline{27}_{16}\overline{27}_{16} \tag{2}$$

We have chosen only a subset of allowed terms in the theory and believe that it is reasonable to assume a discrete symmetry (perhaps in the context of a string model) that would allow only this subset. In any case since we are dealing with a supersymmetric theory, radiative corrections will not generate any new terms in the superpotential.

Note that in Eq. (2), since it is the first term that leads to lepton and quark masses of various generations, it carries a generation label and obeys a hierarchical pattern, whereas the f_i's not being connected to known fermion masses need not obey a hierarchical pattern. We will from now on assume that each $f_i \approx 1$, and see where it leads us.

After substituting the VEVs for the Higgs fields in the above equation, we find a 5×5 mass matrix[2] of the following form for the neutral lepton fields of each generation in the basis $(\nu, \nu_s, \nu^c, E_u^0, E_d^0)$:

$$M \;=\; \begin{pmatrix} 0 & 0 & \lambda_i v_u & f_i v' & 0 \\ 0 & 0 & 0 & \lambda_i v_d & \lambda_i v_u \\ \lambda_i v_u & 0 & M_{\nu^c,i} & 0 & 0 \\ f_i v' & \lambda_i v_d & 0 & 0 & M_{10,i} \\ 0 & \lambda_i v_u & 0 & M_{10,i} & 0 \end{pmatrix} \tag{3}$$

Here $M_{\nu^c,i}$ is the mass of the right handed neutrino and $M_{10,i}$ is the mass of the entire **10**-plet in the **27** matter multiplet. Since **10** contains two full SU(5) multiplets, gauge coupling unification will not be effected even though its mass is below the GUT scale.

Note that the 3×3 mass matrix involving the (ν^c, E_u^0, E_d^0) have superheavy entries and will therefore decouple at low energies. Their effects on the spectrum of the light neutrinos will be dictated by the seesaw mechanism [14]. The light neutrino mass matrix involving ν_i, ν_{is} can be written down as:

$$M_{light} \simeq \frac{1}{M_{\nu^c,i}} \begin{pmatrix} \lambda_i v_u & f_i v' & 0 \\ 0 & \lambda_i v_d & \lambda_i v_u \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 0 \end{pmatrix} \begin{pmatrix} \lambda_i v_u & 0 \\ f_i v' & \lambda_i v_d \\ 0 & \lambda_i v_u \end{pmatrix} \tag{4}$$

---

[2]Although the form of this mass matrix is same as in [10], the results of our paper are different.

where $\epsilon_i = M_{10,i}/M_{\nu^c,i}$. Note that ϵ_i is expected to be of order one. This leads to the 2×2 mass matrix for the (ν, ν_c) fields of each generation which is of the form in Eq. (1),

$$M_i = m_{0i} \begin{pmatrix} \lambda_i^2 & \lambda_i \bar{f}_i \\ \lambda_i \bar{f}_i & \lambda_i^2 \bar{\epsilon}_i \end{pmatrix} \tag{5}$$

Here $m_{0i} = \frac{v_u^2}{M_{\nu^c,i}}$, $\bar{f}_i = f_i \epsilon_i v'/v_u$, and $\bar{\epsilon}_i = 2\epsilon_i cot\beta$. Taking $M_{Pl} \sim 10^{19} GeV$, $M_{GUT} \sim 10^{16}$ and reasonable values of the unknown parameters e.g. $\alpha_i \approx 0.1$, $\gamma_i \approx 0.1$, $f_i \approx 1$, $v' \approx v_u$, we get $m_{0i} \simeq 20$ eV and $\epsilon \approx 1$ which leads us to the desired pattern of masses and mass differences outlined in the introduction.

A crucial assumption in our analysis is that that one of the Higgs fields has a vev along ν^c direction with a low scale (the v' above). We will now demonstrate what kind of a superpotential can lead to such a situation.

Consider

$$W = M 27' \overline{27}' + S \overline{27}' 27_{16} \tag{6}$$

Since 27_{16} has a VEV, this implies that one linear combination of S and the ν^c component of $27'$ (denoted by S') remains light while everything else in $27'$ and $\overline{27}'$ become heavy. If in addition the superpotential contains the couplings

$$W = S 27_{10} \overline{27}_{10} + S^3 \tag{7}$$

since 27_{10} and $\overline{27}_{10}$ have electroweak scale VEVs the light combination of S and $27'$ (S') also picks up an electroweak scale VEV from the trilinear soft supersymmetry breaking terms. Note that this is inevitable as long as electroweak symmetry is broken because such a trilinear term then becomes a linear term in the potential for S' and hence S' must pick up a VEV. We thus see that it is possible to get vev for the singlet field ν^c in the desired **27**-plet of the order of the electroweak scale.

Let us next address the question of the generation mixing. We will assume that it parallels that in the quark sector i.e. the mixing angles to start with are small. Since the neutrino mixings have an additional contribution coming from their seesaw mechanism, we can easily have them be smaller than the corresponding quark mixings. This is for instance what one would like in order to fit the LSND data. We do not get into the details of this since clearly it does not effect the main point of the paper.

Let us end with a few comments on the phenomenological and cosmological implications of the model. The most severe test of this model will come from the understanding of big bang nucleosynthesis [15]. Our model within the standard assumptions that go into the discussion of BBN would imply $N_\nu = 6$ i.e. three extra neutrinos. However, in models with sterile neutrinos, possibilities of large lepton asymmetry at the BBN era has been discussed [16].

The second point that needs emphasizing is that in our model, both the solar and atmospheric neutrinos involve separate sterile neutrinos in the final state. There are well known tests [17] of such models for the atmospheric neutrino oscillations [18] where one looks for neutral pion production. For solar neutrinos, our model is testable by the neutral current measurement planned for the SNO experiment [19].

In conclusion, in this paper we have pointed out a simple way to understand theoretically challenging possibility of a tiny mass difference squared that may arise if the solar neutrino puzzle is to be solved via the vacuum oscillation solution. We exploit an apparent symmetry between the solar and the atmospheric case arising from the maximality of mixing angles to suggest that the ultra small Δm^2_{solar} may be undestandable in models of E_6 type, which automatically contain a sterile neutrino in each **27** that also contains other known particles of each generation and where the generational neutrino mass difference squared may be proportional to the cube of the lepton Yukawa couplings. In this model we can also accomodate the indication for neutrino oscillations from LSND.

This work has been supported by the National Science Foundation grant under no. PHY-9802551 .

REFERENCES

[1] Super-Kamiokande collaboration, Y. Fukuda et al. Phys. Rev. Lett. **81**, 1562 (1998).

[2] B. T. Cleveland et al. Nucl. Phys. B(Proc. Suppl.) **38**, 47 (1995); K.S. Hirata et al., Phys. Rev. **44**, 2241 (1991); GALLEX Collaboration, Phys. Lett. **B388**, 384 (1996); J. N. Abdurashitov et al., Phys. Rev. Lett. **77**, 4708 (1996).

[3] N. Hata and P. Langacker, Phys. Rev. **D56**, 6107 (1997); J. Bahcall, P. Krastev and A. Y. Smirnov, Phys. Rev. **D 58**, 096016 (1998).

[4] Y. Fukuda et al. Phys. Rev. Lett. **82**, 1810 (1999).

[5] M. C. Gonzales-Garcia et al. hep-ph/9903473.

[6] D. Caldwell and R. N. Mohapatra, Phys. Rev. **D 48**, 3259 (1993); J. Peltoniemi and J. W. F. Valle, Nucl. Phys. **B406**, 409 (1993); S. M. Bilenky, C. Giunti and W. Grimus, hep-ph/9711416; V. Barger, T. Weiler and K. Whisnant, Phys. Lett. **427**, 97 (1998); S. Gibbons, R. N. Mohapatra, S. Nandi and A. Raichoudhuri, Phys. Lett. **430**, 296 (1998); E. Ma and P. Roy, Phys. Rev. **D 52**, R4780 (1995); S. Goswami, Phys. Rev. **D 55**, 2931 (1997); for review and other references, see R. N. Mohapatra, hep-ph/9903261 and S. M. Bilenky, C. Giunti and W. Grimus, to appear in Prog. Part. Nucl. Phys. (1999).

[7] R. Foot, H. Lew, and R. R. Volkas, Mod.Phys.Lett.A7, 2567 (1992); R. Foot, Mod. Phys. Lett. A9, 169 (1994).

[8] Z. Berezhiani and R. N. Mohapatra, Phys. Rev. **D 52**, 6607 (1995); R. Foot and R. Volkas, Phys. Rev. **D 52**, 6595 (1995).

[9] F. Gursey and P. Sikivie, Phys. Rev. Lett. **36**, 775 (1976); P. Ramond, Nucl. Phys. **B210**, 214 (1976).

[10] E. Ma, Phys. Lett. **B 30**, 286 (1996).

[11] S. Mohanty, D. P. Roy and U. Sarkar, Phys. Lett. **B 445**, 185 (1998); W. Grimus, R. Pfeffer, T. Scwetz, hep-ph/9905320; G. Mcglaughlin, J. M. Fetter, B. Balatenkin and G. Fuller, astro-ph/9902106.

[12] C. Athanassopoulos et al. Phys. Rev. Lett. **75**, 2650 (1995); C. Athanassopoulos et al. nucl-ex/9706006.

[13] For some alternative ways to understand this, see E. Ma, hep-ph/9903273; hep-ph/9902392.

[14] M. Gell-Mann, P. Rammond and R. Slansky, in *Supergravity*, eds. D. Freedman *et al.* (North-Holland, Amsterdam, 1980); T. Yanaguida, in proc. KEK workshop, 1979 (unpublished); R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[15] For a recent discussions, see E. Lisi, S. Sarkar and F. L. Villante, hep-ph/9901404; K. A. Olive and D. Thomas, hep-ph/9811444.

[16] R. Foot and R. Volkas, Phys. Rev. **D 55**, 5147 (1997); N. F. Bell, R. Foot and R. Volkas, Phys. Rev. **D 58**, 105010 (1998).

[17] F. Vissani and A. Smirnov, Phys. Lett **B 432** , 376 (1998); K. R. S. Balaji, G. Rajasekaran and S. Uma Sankar, hep-ph/9806256.

[18] R. Foot, R. Volkas and O. Yasuda, Phys. Rev. **D 57**, 1345 (1997); Q. Liu and A. Y. Smirnov, Nucl. Phys. **B 524**, 505 (1998);J. Bowes and R. R. Volkas, J.Phys.G24, 1249 (1998); A. Geiser, Phys. Lett. B444, 358 (1999); Y. Koide, Phys. Rev. D59, 053004 (1999).

[19] J. Klein, Review talk at the *Neutrino Telescope* conference, (1999).